



Examiners' Report Principal Examiner Feedback

January 2021

Pearson Edexcel International Advanced
Level

In Further Pure 1 (WFM01/01)

Paper 1: Further Pure F1

General

This paper was accessible, with most candidates finding plenty of opportunities to apply their knowledge and understanding of the topics in this paper. The most challenging topics were induction and coordinate geometry, with the work on geometric transformations and complex numbers providing some challenge to discriminate between higher grades too.

Questions 1 through to 6 each provided openings for candidate of all abilities, testing many standard ideas and techniques. Question 9(i) proved very challenging, though the more standard induction in 9(ii) did provide an opening for most candidates. Question 7 and Question 8 provided a challenge to higher grade candidates.

Report on individual questions

Question 1

In part (a), the vast majority of the candidates worked in radians and correctly evaluated $f(0.2)$ and $f(0.6)$, with the occasional evaluation being found in degrees. However, the A mark was often not scored, indeed it proved to be the hardest mark on the paper to attain, due to candidates not stating that $f(x)$ has to be continuous to justify the existence of the root. That there is a change of sign alone was not sufficient. It is not necessary to show that the function is continuous, simply to say that it must be continuous in the given interval. The lack of mention of continuity has been condoned on previous occasions, but it is something that candidates ought to appreciate is needed.

In part (b), the correct interval was found by the majority of the cohort with the interval bisection method being well rehearsed and understood. A number of candidates used linear interpolation and as in part (a), the use of degrees was sometimes seen. Some candidates lost the final mark because they failed to state the interval. A few stated the wrong interval for their values.

Question 2

This question was well answered and accessible to most candidates with many gaining full marks.

Almost all candidates gained the B1 mark in part (a). Of those who did not it was generally down to not answering the question at all.

In part (b) the most common approach was to multiply out two factors containing the complex root, $(x - \text{root})(x - \text{root})$, to obtain a quadratic. Most candidates did this successfully. A few arithmetic and sign errors were seen when combining terms, the most common being when finding the product of the roots. A fairly common error was to evaluate the product as $\frac{71-9}{64}$

A few candidates attempted, generally successfully, to form a quadratic using the $x^2 - (\text{Sum of roots})x + (\text{Product of roots})$ rule. Having found a quadratic, most then went on to multiply this by $(x - 4)$, or multiply out the terms required to find p and q . Often the step of multiplying through by 4 was stated as an equality of expressions rather than considering the equation, but this was overlooked. Candidates should nevertheless be careful to ensure they have mathematically correct statements. A few candidates forgot to multiply through by 4 entirely and so lost the final A mark.

Some candidates attempted long division of the quadratic into the cubic and were mainly successful.

A few candidates attempted the factor theorem. In most cases they only got as far as finding an equation in p and q and then did not know how to proceed, failing to generate two equations in two unknowns.

A handful of candidates went on to form an equation using a complex root, equate real and imaginary parts and proceed to a correct answer. This however was not the most efficient method.

A number of candidates attempted dividing the cubic by $(x - 4)$ obtaining a remainder and a quadratic quotient. Most managed to reach a correct quotient and a remainder in terms of p and q , but sign slips were quite common when attempting to find p and q .

Question 3

This question was very well answered with a majority of candidates scoring all 4 marks.

- (a) The most common method here was to identify the determinant correctly, set it equal to 0 and then find the required values of k using factorisation. A small number of candidates used the quadratic formula to identify the values of k . The most common error was to fail to deal with the minus signs in the matrix correctly thus finding $\det M = k^2 + 5k + 6$. However, most of these candidates did set the determinant equal to 0 and solved so they were able to gain M1A0.
- (b) This part was very well attempted with the process of finding the inverse of a 2x2 matrix being understood. A very small number of candidates did not appear to know how to form the matrix of signed minors but were aware that they need to use $\frac{1}{\det} \times$. Other candidates found the matrix of signed minors then did not how to use the determinant, giving answers such as

$$\det M = (k^2 + 5k - 6) \begin{pmatrix} k & 2 \\ 3 & k+5 \end{pmatrix}.$$

Unusually, a significant number of candidates used the determinant as $k(k+5) - 6$ in their inverse, and a few used $(k-1)(k+6)$.

Question 4

Another very well answered question with only the final mark falling below and 80% success rate.

In part (a), the required values of $\alpha + \beta$ and $\alpha\beta$ were correctly found by over 95% of the candidates, most of whom then went on to use a correct identity to find the value of $\alpha^3 + \beta^3$. Sometimes this was first by finding the value of $\alpha^2 + \beta^2$ before finding the sum of cubes. A few failed to use the correct identity, with $(\alpha + \beta)^3 - 2\alpha\beta(\alpha + \beta)$ being a common error in such cases.

The number of algebraic errors here was extremely small and it is pleasing to note that there were very few candidates who solved the given quadratic equation.

In part (b), the method of determining a quadratic equation with related roots was well understood and executed by most candidates. The first 3 marks were generally achieved with only a few mistakes, mainly numerical slips. Some made hard work of the B mark, not recognising that the product of roots was simply $\alpha\beta$. A few attempts of expanding the quadratic with correct factors were seen – although a more complicated approach it still usually yielded successful results. Most did gain the final M mark which was encouraging; the method seemed better understood than in previous sessions. If the final A mark had not been lost as a result of earlier slips then it was usually achieved, though as noted, this mark did dip below an 80% success rate. Very few candidates missed the “= 0”, giving their final answer as a function or expression.

Question 5

A routine proof in part (a) gave access to most students with a higher proportion of full mark responses, and the method of (b) also proved to be well known by many.

In part (a) candidates knew they needed to expand the expression and most went on to use 3 correct summation formulas. Unlike some previous series, where a significant number of candidates were unable to make progress in this type of question due to the error of using $\sum 5 = 5$ rather than $\sum 5 = 5n$, in this series there was very little evidence of this error. The final term, $5n$, was usually given correctly. Many were also successful with the algebraic manipulation needed to reach the correct, required form. Some omitted an intermediate step prior to reaching the given answer at the end and consequently lost the final mark. It is important to provide adequate evidence in this type of question

For part (b) most candidates realised that they needed to consider the sum to $2n$ terms and subtract the sum to n terms. There was little confusion with what was required here. Only a

very small number attempted to use $\sum_{n+1}^{2n} = \sum_1^{2n} - \sum_1^{n+1}$ or similar. A handful added instead of subtracting.

Of those who adopted a correct approach, some made extra work for themselves by reproducing all the working for the sum to $2n$ when they could have just substituted $2n$ into their result from part (a). For those who did score the first mark, the remaining challenge was the algebra needed to reach the solution. Many did not recognise the out factors of $n/6$ and $(2n+7)$ [the method in

the mark scheme] instead multiplying out to obtain a cubic which they then had to factorise, albeit generally successfully. A few others took out a factor of $7n/6$ right from the start which left them needing to factorise a long expression involving fractions, though again most achieved it.

Question 6

This proved to be a more demanding question than the earlier ones on the paper, though still largely done well.

Part (a) was accessible to most candidates. Candidates were familiar with the method to find the modulus of a complex number, and most answered the question to give a positive value for λ . However, few spotted the Pythagorean triple, and many took several lines of working, with the occasional careless error, to arrive at an answer.

Part (b) proved to be more challenging for many candidates, although nearly 75% obtained the correct result. However, there were some candidates who seemed unfamiliar with the concept of an argument and so could not answer this part of the question. Amongst those that made progress, most realised they needed to use $\arctan \frac{3}{4}$. A common error was to assume the argument was either $\arctan \frac{3}{4}$ or $\arctan \frac{-3}{4}$. Those in error mostly found the acute angle and did not then subtract from π , failing to identify the correct quadrant. Those who used a diagram tended to have more success, although a correct diagram did not always lead to a correct argument. Some candidates used $\tan\left(\frac{3}{4}\right)$ rather than \arctan .

Many approaches were seen in part (c)(i), with 80% or so scoring all three marks. The most successful were when the candidate substituted for z initially, simplified the $z + 3i$ to $-4 + 6i$ and then proceeded to multiply by the conjugate. The majority found the correct solution, usually multiplying top and bottom by $2+4i$ or $1+2i$ after simplification. Other, less efficient, approaches were to multiply the numerator by the conjugate in terms of z , then substitute for z at a later point. This method was more likely to lead to arithmetic or sign slips, and a few of the candidates did not know what to do with z after multiplying by the conjugate. In general, it was pleasing to see accurate manipulation of the terms and many fully correct solutions.

Part (c)(ii) was another well answered question, with 87% successfully completing it. Candidates seemed familiar with the process of multiplying out the terms and using $i^2 = -1$.. There were some arithmetic or sign errors when gathering terms. Marks were still available in part (d) for such candidates.

Most candidates were able to place the 4 complex numbers into the correct quadrants, and only a few not attempt this part. Some candidates were more concerned with plotting A , B , C and D to scale rather than focussing on their relative positions. This often meant that point C in particular was very close to the origin and difficult to plot accurately, and candidates did not realise that it should be plotted between B and the negative real axis. A few candidates incorrectly plotted A and B in the first and second or the first and fourth quadrants but by far the most common error was to plot C between B and the negative imaginary axis.

Question 7

This question discriminated well and proved to be the second most challenging on the paper. There were some excellent solutions, but a significant number of candidates did not make much progress throughout.

Part (a) was generally well done, though even here less than 90% accessed the first mark (yet was the best of each part in this question), only question 9 having worse starting access. Most candidates seemed familiar with the determinant as a scale factor, though there were some errors in evaluating the determinant. However, a few divided the area by their determinant instead, or used the square of the determinant.

Very few gave a negative answer, but the most common error was in finding the determinant; usually “ $8 + 15$ ”. A small minority of candidates did not know how to make progress and multiplied the matrix by a scale factor 23.

Many candidates gained full marks in part (b) with the accuracy mark for this part being the most successfully achieved final A mark in this questions parts. The most common approach was to multiply A by $\begin{pmatrix} 3p+2 \\ 2p-1 \end{pmatrix}$, equate to $\begin{pmatrix} 17 \\ -18 \end{pmatrix}$ and solve an equation to find p . Many checked both terms. A common error here was to multiply the wrong way round.

It was less common to see $\begin{pmatrix} 17 \\ -18 \end{pmatrix}$ multiplied by the inverse of **A**. When a correct method was applied, errors tended to occur due to their inverse being incorrect.

Though most candidates realised that the matrix **B** represented a rotation in part (c), about 25% thought it was a reflection, so gained no marks. Those who realised it was a rotation usually then gave a full and correct description including centre, direction and angle. The common error when they went wrong was an incorrect angle (or direction), such as 270 clockwise or 90 anticlockwise or angle of 45 degrees. Most did specify the centre of rotation. There were occasional uses of -90 anticlockwise, which gained the mark despite being somewhat unconventional.

Part (d) discriminated well. Only two thirds obtained a fully correct solution, most of whom used the inverse approach, only a small minority evaluated **CA** and solving equations. Most candidates attempted \mathbf{A}^{-1} and attempted to use it. Many evaluated \mathbf{BA}^{-1} to obtain the solution, and the errors where it went wrong were equally split between an incorrect inverse matrix or evaluating $\mathbf{A}^{-1}\mathbf{B}$. A small number of candidates incorrectly multiplied **A** and **B**, gaining no marks.

Question 8

This proved another challenging question, though slightly more accessible than question 7 overall, and mainly accuracy errors causing loss of marks. Roughly two thirds successfully achieved the correct final answers.

In part (a), when finding the intersection point, the most common method was first to find the Cartesian equation of the parabola, $y^2 = 40x$, and then solve this simultaneously with the Cartesian equation of the hyperbola to obtain x or y . Some made slips in this process, but it was usually carried out successfully. The alternative method was to use the parameter t , to form the equation $10t \cdot 20t^2 = 25$. Candidates almost all then obtained the correct coordinates. Some few gave answers with no working, and so did not use algebra so scored no marks.

In general part (b), being a standard question on this specification, was well answered with all the requisite working shown. However, only 80% managed a correct derivative, with 70% reaching the correct equation. When finding the gradient of the parabola, most opted to use the

Cartesian equation $y^2 = 40x$ (or their incorrect version of this), with parametric approaches being less common. With the Cartesian equation they could differentiate implicitly, work via dx/dy , or take the square root to get $y = f(x)$ before finding dy/dx . The last of these was a popular approach but was not always managed without slips. A common error was to think that \sqrt{x} was x^{-1} . Some mistakenly used the wrong curve altogether and found dy/dx for the hyperbola. There were several candidates who tried to find the gradient without differentiating, often treating the value of t at $(10, 20)$ as the gradient of the tangent. This sometimes gave spurious “correct” results. Most knew how to find the normal gradient and hence a normal equation. Some lost the final mark because they failed to give the answer in the form required but to leave it as $x + y = 30$ or $y = 30 - x$.

In part (c) most knew that they needed to solve simultaneously using their normal equation and the parabola. Again, the parametric form of the parabola was less commonly used. Despite all the possible pitfalls on the way, correct answers were common. Even those candidates who had errors in part (b) were able to gain the Method marks here through careful use of the required algebra. It was evident though that many candidates were relying on their calculators to solve their 3TQ rather than clearly showing all the steps in their solution. This choice proved costly for those candidates who, following an incorrect answer to part (b), obtained incorrect roots with no method shown. Again, a small number simply stated answers with no working and were consequently unable to access the marks.

Question 9

Induction remains the most challenging topic on the specification for F1. Part (ii) was generally attempted more successfully than part (i), though full marks was actually slightly more common in part (i) than in part (ii).

In part (i), as is often the case in using induction to prove a recurrence formula, a significant minority of the cohort, in finding u_2 , did not prove that the result is true for $n = 1$. They often evaluated for $n=2$ believing they were evaluating for $n=1$. Others evaluated for more values than necessary in this context. The subsequent substitution of the u_k formula into the u_{k+1} formula and simplification were often done correctly with the associated algebra handled well. The required descriptive details were of a mixed quality. The language for setting up the induction for example “assume $n=k$ ” rather than “assume the result is true for $n=k$ ” tended to

highlight a general lack of fluency with the technique. Major losses of marks tended to be for failing to understand which relationships to use and simply failing to show that u_k is true $\Rightarrow u_{k+1}$ is true. A small but significant number confused index values for example dealing with u_{k+2} in terms of u_{k+1} or even u_k , although this could earn marks for correct algebra. Most candidates included the four key concluding steps necessary in any proof by induction, whether or not they had produced correct work previously but only 50% scored the final A.

In part (ii), the vast majority of candidates showed that the result is true for $n = 1$, then went on to use either $f(k+1) - f(k)$ or $f(k+1)$ alone in some way. Over 90% of candidate accessed these two marks. Some, however, stopped there, with only 75% progressing to attempt to prove that the result is true for $n = k + 1$ on assuming true for $n = k$. Some more sophisticated approaches involved subtracting $2f(k)$, $9f(k)$, etc. which led to easier simplification of the algebraic terms. This generally scored the first 3 marks. The main slips in the working here involved incorrectly expressing terms involving 2^{k+3} and 3^{2k+3} in terms of 2^{k+2} and 3^{2k+1} although the quality of the work shown in this process has improved from previous years. Many candidates lost the final 2 marks because they failed to find an expression for $f(k+1)$ that was clearly divisible by 7. It was also acceptable to construct an argument based on saying that, for example, if $f(k+1) - f(k)$ is divisible by 7 then $f(k+1)$ is divisible by 7. Again, the standard of the descriptive conclusions was mixed, with several candidates omitting the conclusion that $f(n)$ is divisible for all n . Only 43% successfully score the final mark.